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Experiment and analysis for non-Fourier conduction in materials with non-homogeneous inner structure

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Dedicated to Professor Dr.-Ing. habil. Thaddaeus M. Bes on the occasion of his 65th birthday

Abstract

The proposition of hyperbolic conduction (also known as the second sound wave) for materials with non-homogeneous inner structure has run into a serious controversy in recent times. While one group of investigators has observed very strong evidence of hyperbolic nature of conduction in such materials and experimentally determined the corresponding relaxation times to be of the order of tens of seconds, the other group proclaims that their experiments do not show any such relaxation behaviour and the conventional Fourier law of conduction is good enough to describe conduction in them. This paper is an effort towards resolving this controversy. In the first place the experimental philosophies and techniques of both the groups have been thoroughly examined. It has been observed that determination of thermophysical properties independent of the relaxation time measurement is an inherent inconsistency in all these experiments. Additionally the assumptions regarding temperature input might have also played a role to arrive at diverging conclusions. Based on these observations an experimental method has been suggested in this study which uses temperature oscillation in semi infinite medium to determine the thermal diffusivity and the relaxation time simultaneously from a single experiment. Using this technique the wide range of experiments conducted reveal that there exists a definite hyperbolic effect in the "bulk" conduction behaviour of such materials although it is somewhat less in extent to those reported by investigators claiming existence of hyperbolic conduction. Also a wide range of experiments with variation of parameters such as packing material, its particle size, filling gas used and its pressure and temperature have been conducted. The data presented here for the wide range of parameters can be useful for further investigations and plausible explanation of "bulk conduction" in materials with non-homogeneous inner structure.

1. Introduction

Hyperbolic or non-Fourier heat conduction in material has been a theme of investigation for considerable time in the recent past. The fundamental ground for this proposition was laid by Maxwell [1] and Morse and Feshbach [2]. However, the formulation in the presently used form of flux equation for non-Fourier conduction can be attributed to Vernotte [3] and Cataneo [4] to give the unsteady flux equation of

$$q+\tau\frac{\partial q}{\partial t}=-\lambda\nabla T \tag{1}$$

where

$$\tau=\frac{a}{C^2}$$

This equation is often referred to in literature as Cattaneo–Vernotte equation. This results in an extended equation for energy balance during conduction in the form

$$\frac{\partial T}{\partial t}+\tau\frac{\partial^2 T}{\partial t^2}=a\nabla^2 T \tag{2}$$

This removes one of the theoretical inconsistencies of the original Fourier's diffusive law which actually points out an infinite propagation velocity for the heat wave meaning that a change in the temperature at any part of the medium should perturb the temperature of a finite medium at each point instantaneously. Due to its similarity with the acoustic wave this proposed wave like propagation of thermal signals is also termed as the "second sound wave". Chester [5] accepted this name as described by Peshkov [6].

However, it is incorrect to say that this theoretical inconsistency was the seed of the development of the

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Nomenclature

a	thermal diffusivity	$m^2 \cdot s^{-1}$
B	amplitude ratio	
C	second sound velocity	$m \cdot s^{-1}$
C_p	isobaric specific heat capacity	$kJ \cdot kg^{-1} \cdot K^{-1}$
G	phase shift	
i	complex number	
q	heat flux	$W \cdot m^{-2}$
T	temperature	K
t	time	s
x	space coordinate	m

Greek symbols

ξ	dimensionless function of space coordinates	
κ	constant	
λ	thermal conductivity	$W \cdot m^{-1} \cdot K^{-1}$
ω	constant angular frequency	s^{-1}
τ	relaxation time	s
Θ	temperature difference	K
Θ_0	complex amplitude	K

Subscript

m	mean

hyperbolic theory of heat conduction. History shows that the development is just the other way round. In fact the finite propagation velocity heat wave in liquid helium at 1.4 K was measured to be 19 $m \cdot s^{-1}$ by Peshkov [6] much before the proposition by Cattaneo and Vernotte [3,4]. This is quite logical because only micro level physics based correlations (such as the law of radiation or laws resulting from quantum physics) can be predicted theoretically while the constitutive correlations in areas like heat transfer, elasticity or fluid mechanics are observed laws and can begin and end (in terms of proving them) in experiments only. Brown et al. [7] reconfirmed this proposition experimentally which in the last three decades has produced a stream of theoretical analysis such as those by Wiggert [8], Baumeister and Hamill [9], Yuen and Lee [10] and many others which have been very comprehensively reviewed by Özisik and Zhou [11]. Engineering investigations are mostly triggered by applications and the present explosion of highly transient and high heat flux devices such as LASER or computing chips have acted as this trigger. Numerous papers dealing with such and other applications [12–17] have appeared in literature in the recent past. With more emphasis on precision material processing and operation with the above applications it is likely that an increase in the application of hyperbolic proposition will take place in future.

While little doubt exists on the applicability of hyperbolic proposition of heat transfer to the fields such as near absolute zero temperature or highly transient—high heat flux devices (the only question being whether the precision in temporal scale of the effect is required for the particular application), there is another important application area for hyperbolic conduction which has run into considerable controversy even with respect to the existence of a significant hyperbolic effect. It concerns the materials with "non-homogeneous" inner structures. Luikov [18], Brazhnikov [19] and many others reported relaxation time, τ of the order of (up to) 30 s in materials such as meat products while the corresponding values for homogeneous solid, liquid or gases range from 10^{-8} to 10^{-12} s. It may be noted that the relaxation time of homogeneous solids is either the fundamental electron or phonon relaxation time, for gases it is mostly molecular collision relaxation time, etc; however, the relaxation time of non-homogeneous materials with inner structure is an effective relaxation time that is defined through experimental results via the non-Fourier model selected.

To resolve this anomalously large order of magnitude of the relaxation behaviour, two important studies were conducted in the past which concluded in favour of such relaxation behaviour. The most quoted work in this direction was by Kaminski [20], where by measuring a and C the author estimated the relaxation time in some chemicals, sand, glass beads and ion exchanger to be of the order of 10 to 54 s. He also provided a physical explanation of τ and a and a proof of hyperbolic behaviour in these materials in terms of local temperature distribution. The other important investigation which confirmed this result was by Mitra et al. [21]. The material for this investigation was processed meat (Bologna). It was argued that the non-homogeneous inner structure in the form of biological tissues gave rise to hyperbolic behaviour for heat conduction in such a material. They carried out a series of four different types of experiments for which the main aim was to bring the samples of different temperature in contact suddenly and thus observe the propagation of thermal disturbance with a finite velocity which was obvious from each of the experiments. It was found that the thermal relaxation time for such material is of the order of 15 s which, in fact reconfirms the order of magnitude of the values obtained by Kaminski [20]. It was also shown in [21] that the behaviour of processed meat sample cannot be explained by the usual proposition of contact thermal resistance.

However, the above claims of hyperbolic behaviour in materials with non-homogeneous inner structure was strongly confronted by Graßmann and Peters [22] as well as Herwig and Beckers [23]. Both studies reported that in materials with non-homogeneous inner structure no evidence of hyperbolic conduction exists and the conventional Fourier law is good enough to describe transient conduction behaviour in them. For experiment Graßmann and Peters [22] used a cylindrical sample geometry with an axial electrical line heating element and Herwig and Beckers [23] used embedded pipe carrying hot water as heating element in a rectangular box containing the sample. The widely diverging observations of [22] and [23] from that of [20] and [21] have cast a severe doubt on the claims and counter claims on hyperbolic nature of conduction in materials with non-homogeneous inner structure.

The above controversy is needed to be resolved "either way" as mentioned in [23]. However, the study in [23] could not conclusively resolve this controversy "their way" since it could neither justify its experimental philosophy, nor negate the experimental philosophy of Kaminski [20] or Mitra et al. [21] nor did it try to locate the flaw in the theoretical explanation given in [20] and [21]. On the other hand Graßmann and Peters [22] honestly confessed that their finding could not explain why their experimental results confirms the Fourier law which has got a serious theoretical inconsistency. They [22] invited more experimental work to resolve this inconsistency. This is the inspiration of the present investigation. Instead of designing another set of experiment in a subjective way, first the philosophies of experiments conducted earlier [20–23] have been examined here and it was found that some serious inconsistencies exist in the design and procedure of these experiments. Based on this examination, an experimental technique is devised which is devoid of these inconsistencies and measurements are presented for a wide range of materials with non-homogeneous inner structure. The effect of different parameters such as the filling gas, its pressure and temperature as well as the particle size of solid on the thermal diffusion and hyperbolic relaxation behaviour have been presented. The experimental philosophy presented here can give rise to more experimental works with different techniques which can help to resolve this important controversy existing in literature. Also, the wide range of data with various parameters can be useful for comparing measurements required to confirm one theory or the other.

2. Different aspects of experimental philosophy

While all the experimental works [20–23] were aimed at proving or disproving hyperbolic nature of conduction in materials with non-homogeneous inner structure, only one (Mitra et al. [21]) has focused somewhat shortly on the meaning of bulk conduction in such material in terms of "micro" and "macro" effect in its conclusion section. We will go to that aspect later on but at present it is worth looking at the experimental philosophy employed by the above investigators which will show inconsistencies due to lack of clarity of physical meaning of this bulk conduction. Let us consider the experiments one by one.

2.1. Kaminski [20]

While an effort has been made to present physical explanation of τ, the relaxation time, the effort made to reveal the meaning of τ for materials with non-homogeneous inner structure was rather vague depicting it as "structural heat transfer interaction at different level". This has resulted in too much emphasis on penetration depth in the measurement philosophy and neglected the aspect of meaning and method for the determination of thermal diffusivity. This has been discussed by Graßmann and Peters [22], who pointed out that Kaminski's [20] reference of thermal conductivity measuring equation was at least doubtful if not totally incorrect. They [22] have also indicated at a possible confusion between a and λ which is difficult to check in such experimental philosophy where determination of a (or λ) is done independent of the relaxation behaviour. At this point let us ask a fundamental question. How can one determine thermal diffusivity (or conductivity) of a material independent of its relaxation behaviour while the basic constitutive relation which defines thermal conductivity has been changed to a relaxation related one (Eq. (1))? It is terribly inconsistent to use the equations and methods given in Carslaw and Jeager [24] which are based on Fourier conduction to describe conductivity which is argued to show hyperbolic behaviour.

2.2. Mitra et al. [21]

This particular study performed more organized experiments, so logically the evidences of the existence of hyperbolic conduction in processed meat they presented looks more reasonable. Depending on the nature and relative directions of wave propagation, different experiments were devised. While the claim of the temperature jump they observed and the experimental sensitiveness they reported are quite consistent, no clear reason to refute their experiment could be presented by the authors opposing the hyperbolic proposition [22,23] except a subjective "doubtful" labelling. These experiments were also preceded by a validating experiment with aluminium slab.

However, like Kaminski [20] the experimental philosophy also suffers from the drawback of determining different thermophysical quantities for the sample from different experiments such as λ from steady state test, c_p from DuPont method and density by sensitive mass balance. Now the determination of a from λ, ρ and c_p measured separately usually gives worse results than determining a directly from transient experiment. A careful examination will reveal that the sensitivities of both a and λ determined by different methods may not be same and steady state experiment can result in a deviation in λ which can offset the result of τ measurement even by an order of magnitude. So the philosophy of assembling λ, c_p, ρ and C determined from different experiments is inconsistent.

2.3. Graßmann and Peter [22]

This study was consistent in the sense that it concentrated more on showing that for wet sand Fourier behaviour exists rather than refuting non-Fourier behaviour. However, it also suffered from the inconsistency that for the computation they presented (Fourier case), they used the value of conductivities, determined through different experiments. What was even more serious flaw in their experiment was that they used an electrical cartridge heater which gives a constant heat flux type boundary condition rather than constant temperature type. They assumed that the heat generating tube embedded in the sample generated a uniform temperature input to the sample identical so that calculated theoretically from the power input. This is a pious thought which is seldom satisfied in actual practice. In fact in such cases an actual input can be observed by recording the temperature in the heating element spatially and temporally. Thus a real picture of delay with respect to input is not possible to obtain by these experiments.

2.4. Herwig and Beckers [23]

In this recent work also like the other authors the thermal diffusivity was determined from fitting curve with a separate experiment based on Fourier law. Although directly a has been measured here, the problem with its independent determination exists. Moreover the problem with the inlet temperature profile exists here as well. Even though sending hot water will create a better temperature distribution in the wall, it is preferable to use that exact distribution for numerical calculation. Also the experimental results of processed meat shows quite inconsistent oscillation which requires to be explained in order to refute more smooth response presented by Mitra et al. [21]. Probably sampling rate of one measurement per second was too coarse because it cannot capture events of smaller order.

The above discussion on experimental philosophy brings out that there are some inconsistencies in the experimental techniques chosen both by the experiments claiming [20,21] and counter claiming [22,23] the existence of hyperbolic conduction in materials with non-homogeneous inner structure. To overcome these inconsistencies in the present paper an experimental method is devised in which simultaneously a and τ are determined from the same experiment. At the same time the exact recorded boundary temperature history has been used in the theoretical model for data reduction without assuming any ideal input (such as sinusoidal oscillation).

3. Principle of the present experiment: mathematical model

Since the finite propagation velocity of thermal wave is the main theme of investigation, a semi infinite geometry of the test sample is chosen so as to have enough sample length to attenuate the input signal. The input was chosen very carefully to be a periodic oscillation since only for oscillatory inputs the hyperbolic effects can persist after a long time as opposed to instantaneous inputs such as step change used in all the experiments in the references [20–23]. This was clearly demonstrated in the theoretical analysis of Yuen and Lee [10] using a sinusoidal surface heat flux in a semi infinite medium. Due to the above consideration, the conduction in a sample of semi infinite geometry with an oscillatory (known) boundary temperature variation has been chosen. For simultaneous measurement of thermal diffusivity and relaxation time of thermal wave, the oscillatory input has got an additional advantage that the two quantities amplitude attenuation and phase shift can be measured. Thus two unknown quantities can be evaluated explicitly without the need of any iteration or fitting. This basic principle is used to measure a and τ simultaneously from a single experiment which is consistent with the constitutive relation. Additionally surface temperature recorded at the sample boundary is used for modelling. The mathematical model for hyperbolic one dimensional heat transfer in semi infinite medium is given by

$$a\frac{\partial^2 T}{\partial x^2}=\frac{\partial T}{\partial t}+\tau\frac{\partial^2 T}{\partial t^2} \tag{3}$$

For a given oscillatory (sinusoidal) input at the sample surface (finite end) the boundary condition can be given by

$$T(x=0,t)=T_m+(T_0-T_m)\cos(\omega t) \tag{4}$$

Using temperature difference $\Theta = T - T_m$, this condition reduced to

$$\Theta(x=0,t)=\Theta_0\exp(i\omega t) \tag{5}$$

where Θ_0 is the complex amplitude of oscillation.

Similarly, the boundary condition at the infinite side can be depicted as

$$\Theta(x=\infty,t)=0 \tag{6}$$

The initial condition for the entire medium may be (for the experiment)

$$\Theta=0,\quad \text{at } t=0 \tag{7}$$

However, this condition is not needed for the evaluation because steady oscillations are considered.

Eq. (3) can be reduced to the temperature difference form

$$a\frac{\partial^2\Theta}{\partial x^2}=\frac{\partial\Theta}{\partial t}+\tau\frac{\partial^2\Theta}{\partial t^2} \tag{8}$$

This equation along with boundary conditions (5) and (6) can be solved by assuming the solution in the form,

$$\Theta(x,t)=\xi(x)\exp(i\omega t) \tag{9}$$

This form of solution physically implies that the oscillation is propagated in the medium with an amplitude which attenuates with space co-ordinate as well as phase shift.

Using this assumption, we can reduce the differential Eq. (8) to the form

$$\frac{d^2\xi(x)}{dx^2} - \xi(x)\left[\frac{i\omega}{a} - \frac{\tau\omega^2}{a}\right] = 0 \tag{10}$$

This equation has a general solution in the form

$$\begin{aligned}\xi(x) &= C_1 \exp\left(-x\sqrt{\left[\frac{i\omega}{a} - \frac{\tau\omega^2}{a}\right]}\right)\\ &\quad + C_2 \exp\left(x\sqrt{\left[\frac{i\omega}{a} - \frac{\tau\omega^2}{a}\right]}\right)\end{aligned} \tag{11}$$

Putting boundary conditions (5) and (6) yields

$$\begin{aligned}\Theta(x,t) &= \xi(x)\exp(i\omega t)\\ &= \Theta_0 \exp\left(i\omega t - x\sqrt{\left[\frac{i\omega}{a} - \frac{\tau\omega^2}{a}\right]}\right)\end{aligned} \tag{12}$$

Taking the real part as the solution we can have the temperature distribution as

$$\Theta(x,t) = \Theta_0 \exp\left(-x\sqrt{\frac{\omega}{2a}\kappa}\right)\cos\left(\omega t - x\sqrt{\frac{\omega}{2a}\frac{1}{\kappa}}\right) \tag{13}$$

where

$$\kappa = \sqrt{(\tau\omega)^2 + 1} - \tau\omega$$

In the limit $\tau \to 0$ the solution reduces to the well-known solution for Fourier condition in a semi infinite medium with oscillatory boundary condition.

$$\Theta(x,t) = \Theta_0 \exp\left(-x\sqrt{\frac{\omega}{2a}}\right)\cos\left(\omega t - x\sqrt{\frac{\omega}{2a}}\right) \tag{14}$$

From the solution given by Eq. (13), the amplitude ratio and the phase shift can be calculated as

$$B = \frac{\Theta}{\Theta_0} = \exp\left(-x\sqrt{\frac{\omega}{2a}\kappa}\right) \tag{15}$$

and

$$G = x\sqrt{\frac{\omega}{2a}\frac{1}{\kappa}} \tag{16}$$

From this, one can determine thermal diffusivity as

$$a = \frac{\omega}{2}\left[\frac{x}{\ln(B)}\right]^2 \kappa = \frac{\omega}{2}\left[\frac{x}{G}\right]^2 \frac{1}{\kappa} \tag{17}$$

The only unknown parameter κ can be determine as

$$\kappa = \frac{\ln(B)}{G} \tag{18}$$

Thus, the relaxation time can be determined from the value of κ as

$$\tau = \left[1 - \kappa^2\right]\frac{1}{2\omega\kappa} \tag{19}$$

Thus, from the two measured quantities (B and G), both a and τ can be calculated from Eqs. (17) and (19) respectively with the help of Eq. (18). It is interesting to note that both a and τ are some combinations of B and G, thus if by this method we get a consistently good result of a which is supported by literature then it is an indirect proof that the calculated values of τ are also correct because they have been obtained from the same values of B and G which were used to compute a.

4. Experimental set-up and procedure

The experimental cell given in Fig. 1 consists of a cylindrical well bore in a block (1) of POM (polyoxymethylene). The diameter of the bore is 40 mm and the length 80 mm which holds the sample and acts as a semi infinite medium. The cavity is closed on the sides by the removable discs (2) of stainless steel. They act as heating (bottom) and cooling (top) surface, respectively. Between the bottom disc and the water cooling chamber (3) a Peltier element (4) is sandwiched which creates the required temperature oscillation with the help of a function generator attached to the DC supply. The cooling chamber at top and bottom are supplied with water from a constant temperature bath which act as a controller of mean temperature of oscillation. At the interface of the sample material and the bottom disc, a thermocouple (5) of 0.5 mm diameter is located in a groove in the bottom disc having a shape identical as the thermocouple. Another five thermocouple are inserted above this with a distance of 10 mm from each other from the bottom towards the top. The tips of all thermocouples are located at the axis of the cylinder. These thermocouples measure the responses at various locations and also confirm whether semi infinite assumption holds true. The test section has also provisions for gas filling (6) and evacuation (7).

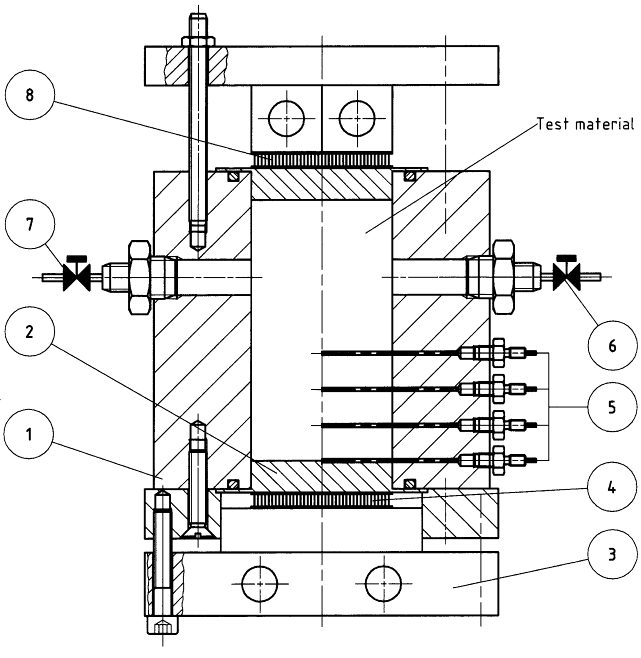

Fig. 1. The test cell for experiment.

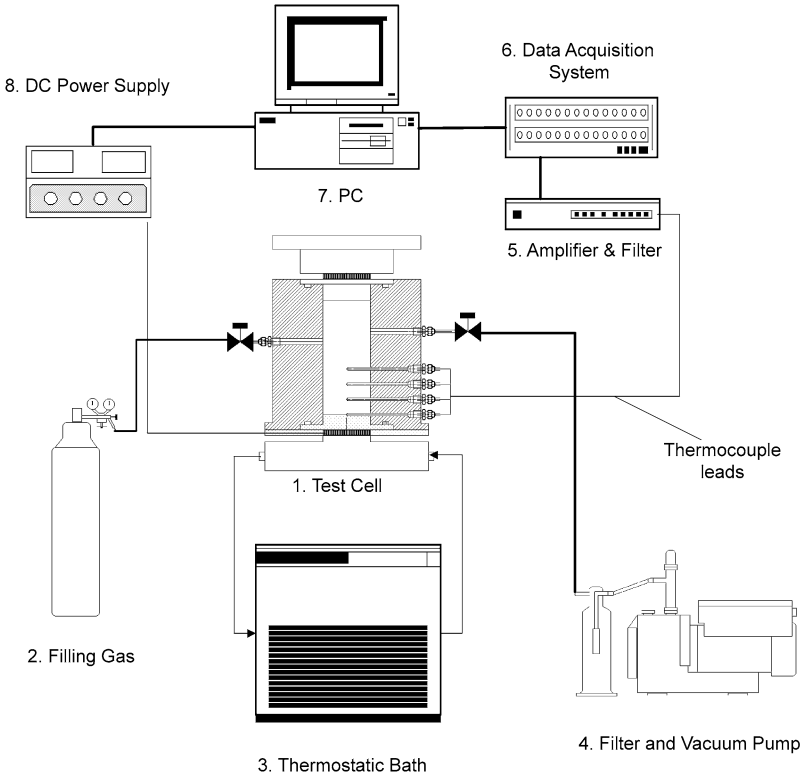

Fig. 2. The circuit of experimental setup.

This cell is connected in an experimental circuit shown in Fig. 2. The cell is fed with cooling water from a thermostatic bath used for temperature control. On one hand a gas cylinder filled with required gas is connected to the experimental cell through a valve on the other hand to evacuate one gas before filling another and also to have sub-atmospheric gas pressure, the cell is connected to a vacuum pump through a filter which traps any solid particle from the sample material from escaping to the vacuum pump during its operation.

The test cell is fitted with pressure gauge to read pressure during both positive and negative gauge pressure operation. The leads from thermocouples (all K type, Chromel-Alumel) are connected to data acquisition system through an amplifier and filter. This data acquisition system is connected to a PC for data recording and storage. The Peltier element is given a DC supply through the function generator which is also triggered from the same PC in order to have a controlled amplitude and frequency of temperature oscillation at the input.

At the beginning of each experiment the thermocouples are inserted and their positions are checked accurately. Then the test cell is filled with solid particles. The particles used are copper coated lead spheres (4.4 mm diameter), synthetic sand (SiO_2), alumina powder of different particles sizes, sodium hydroxide powder and processed meat (Bologna). Subsequently the test cell is evacuated using the vacuum pump. The pressurizing gases used are helium, nitrogen, argon and air. After closing the vacuum line, the valve in the filling line is opened and the gas from the connected cylinder fills in. The evacuation and filling process is repeated to eliminate the possibility of traces of residual gas from the previous experiment. Finally the filling of gas is done at the required pressure. The temperature oscillation is triggered from the PC with pre-assigned voltage setting. However, the fine tuning of the input wave is done through the control of the temperature and flow rate of water from the thermostatic bath used for cooling. The initial oscillations are not used for computation because of their transient nature. Once steady oscillations are reached, the input oscillations along with attenuated and shifted oscillations inside the test media are recorded. Thermocouples far up in the medium show that the oscillations completely die down and thus the approximation of semi infinite medium holds true.

5. Data reduction, sensitivity and error estimates

The data recorded by the acquisition system are the temperatures sensed by the thermocouples at different axial positions of the sample. On the heating side it is not required that the oscillation at the interface of the reference material and the porous sample be taken. In fact this particular reading can give an average between the two and not necessarily temperature of the sample at $x = 0$. The next two thermocouples can be taken with the lower one as input temperature to the sample. A typical response for the five consecutive thermocouples are shown in Fig. 3. It is evident

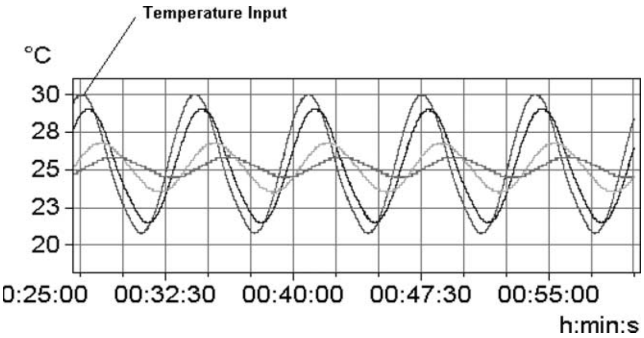

Fig. 3. A typical record of input and attenuated temperature oscillations.

the oscillation has totally died down at the far end and the medium may be treated as a semi infinite medium. These raw oscillation data were then transformed into temperature difference and with the help of a mathematical software the phase shift and amplitude attenuation was determined for the final steady part of oscillation. Usually last ten to fifteen cycles were taken for evaluation. From these data of B and G, the values of a and τ were calculated using Eqs. (17)–(19).

An analysis for sensitivity of the values of a and τ against the uncertainties of measurement in B and G was carried out. It shows that

$$\frac{\Delta a}{a}=2\frac{\Delta x}{x}+\frac{\Delta\omega}{\omega}-\frac{\Delta\kappa}{\kappa}-2\frac{\Delta G}{G} \tag{20}$$

and

$$\frac{\Delta\tau}{\tau}=\left[\frac{\kappa^2+1}{1-\kappa^2}\right]\frac{\Delta\kappa}{\kappa}-\frac{\Delta\omega}{\omega} \tag{21}$$

where

$$\frac{\Delta\kappa}{\kappa}=\frac{\Delta(\ln B)}{\ln B}-\frac{\Delta G}{G} \tag{22}$$

This means that the measurement of relaxation time is more sensitive to the measurement of amplitude attenuation or phase shift than thermal diffusivity. In the present experiment, the thermocouples used have an accuracy of 0.05 °C and their response time is of the order of 50 ms. This corresponds to an uncertainty in the determination of τ as 10% and that of a as 6%. However, since the quantities B and G are not absolute values of temperatures but their ratio and difference of temperatures, the error in them is smaller in magnitude compared to the absolute error in temperature measurement which makes the present accuracy acceptable. It can also be kept in mind that the aim of the present experiments is to determine the "order" of the values of τ to resolve the dispute over the existence of hyperbolic conduction rather than suggesting accurate values for them. The parametric study is more to reveal the trends for these effects which also conforms to the present level of accuracy.

The above errors are systematic errors. The random errors are smaller (rel. error below 1%) and can be neglected in the error analysis.

6. Results and discussion

The results obtained can be broadly divided into two parts. First, experiments with non-homogeneous materials with air as the filling gas under atmospheric pressure in the cavities of solid particles. The solid materials were chosen to be those on which experiments were conducted in the literature [20–23] to prove or disprove hyperbolic conduction. The results are shown in Table 1. It can be seen that present results confirms a hyperbolic behaviour of conduction phenomenon for all these materials with non-homogeneous inner structure even though the level of relaxation time obtained in these experiments are much smaller in magnitude compared to those reported in [20] and [21]. It is interesting to note that in some cases the thermal diffusivity determined in the present case are quite close to those reported in [20–23]. For example for processed meat the present measurements give almost the identical values to those of [21] and [24]. Even the other measurements produced a of the same order. In the present measurement technique both a and τ are different combinations of the phase shift G and amplitude attenuation B, hence a and τ are indirectly related to each other. Thus the accurate determination of a is an indirect proof that the τ determined by present experiments are more reliable than those reported in [20–23]. It can be said that there is a definite proof of hyperbolic conduction in these materials because the present results are not only obtained through experiments which determined a and τ simultaneously but also used the actual recorded temperature oscillation given as input. However, in case of sand the thermal diffusivity suggested by the experiment lies in between those suggested by Kaminski [20] and Herwig et al. [23]. This may be important observation because of the value of τ also lies between the value (20 s in [20] and 0 s in [23]) measured by them. This means the key error which shifted them to the two extreme may be the wrong determination of a.

At this point it will not be irrelevant to discuss the meaning of bulk conduction in materials with non-homogeneous inner structure. Herwig and Beckert [23] indicated that the results of Kaminski [20] and Mitra et al. [21] are much

Table 1
Comparison of measured values of thermal diffusivity (a) and relaxation time (τ)

Material	Thermal diffusivity (10^{-6} $m^2 \cdot s^{-1}$)	Relaxation time (s)	Source
Sand	0.226	2.26	Present experiment
	0.408	20	Kaminski [20]
	0.218	0	Graßmann et al. [23]
	0.169	0	Herwig et al. [24]
$NaHCO_3$	0.185	0.66	Present experiment
	0.31	28.7	Kaminski [20]
Processed meat	0.132	1.77	Present experiment
	0.14	15.5	Mitra et al. [21]
	0.1304	0	Herwig et al. [24]

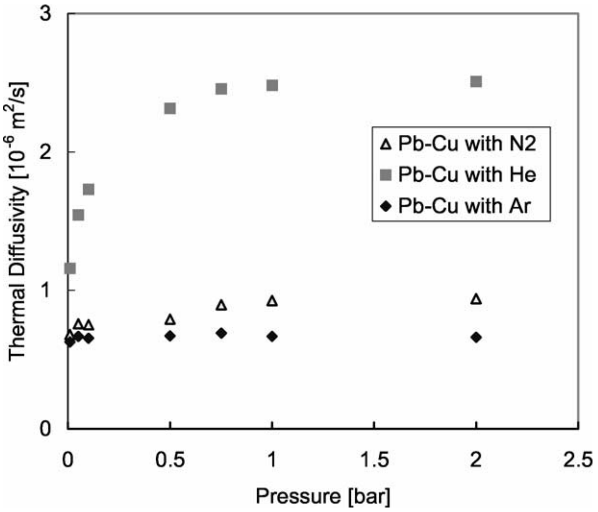

Fig. 4. Effect of filling gas and its pressure on the thermal diffusivity of Cu coated Pb sphere packing.

higher than those talked about by Tzou and Chen [25]. However, they did not look at the fact that conduction in medium with non-homogeneous inner structure is not same as that in the classical sense. It is in fact a combination of phenomena such as conduction, contact resistance, small trace of natural convection and micro level radiation and it is difficult to separate out the effects. They are lumped as a conduction phenomenon which according to present results can be better represented (in a bulk sense) by hyperbolic equation.

The second part of the experiments which was very extensive, was carried out to find the effect of different parameters on the thermal diffusion and relaxation behaviour. Apart from observation of the effects of these parameters, the aim of these experiments was also to provide a large spectrum of experimental data which can used be in future as data base for comparisons.

First, the effect of gas pressure was determined. Fig. 4 shows that at super-atmospheric pressure, the gas pressure does not affect the thermal diffusivity. The corresponding influence on relaxation time is shown in Fig. 5 which also shows very marginal influence. However, both Figs. 4 and 5 show that a considerable influence of pressure exists at the sub-atmospheric pressure range for thermal diffusivity as well as relaxation time. It can be further observed that the extent of influence of pressure and the ranges of values of a and τ critically depend on the filling gas. With lead balls having copper coatings, it is found that the values of a and τ are much higher for helium as the filling gas. The corresponding values for nitrogen and argon are much less in magnitude. It is also noteworthy that with helium as filling gas the effect of pressure in the sub atmospheric range is stronger compared to that with nitrogen and argon, both for thermal diffusivity and relaxation time. When synthetic sand particles are used which are not only different in material from metallic balls but also much smaller in particle size, the above observations held equally true qualitatively with the value of a and τ somewhat shifted as shown in Figs. 6 and 7. These results can be explained by the fact that the filling gas property plays a decisive role for packing in their bulk conduction. It is because of this, helium as filling gas produced a much higher impact on thermal diffusivity. As far as relaxation behaviour is concerned, the wide difference between the relaxation time between metallic balls and sand with same gas has got no physical significance, i.e., the second sound wave velocity is not at the proportion of the relaxation time. Even the increase of relaxation time with pressure is not a universal trend as will be seen later in case of nanoparticles.

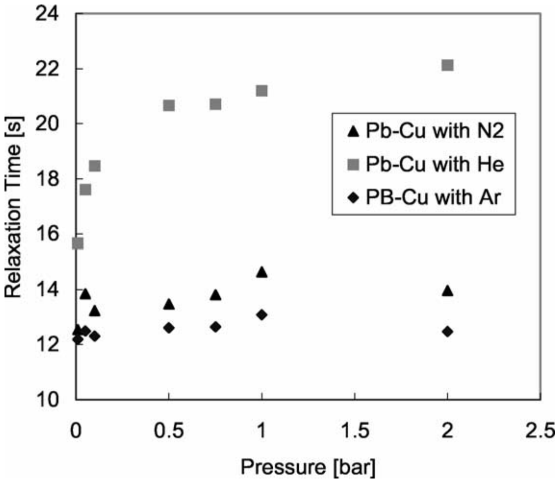

Fig. 5. Effect of filling gas and its pressure on the relaxation time of Cu coated Pb sphere packing.

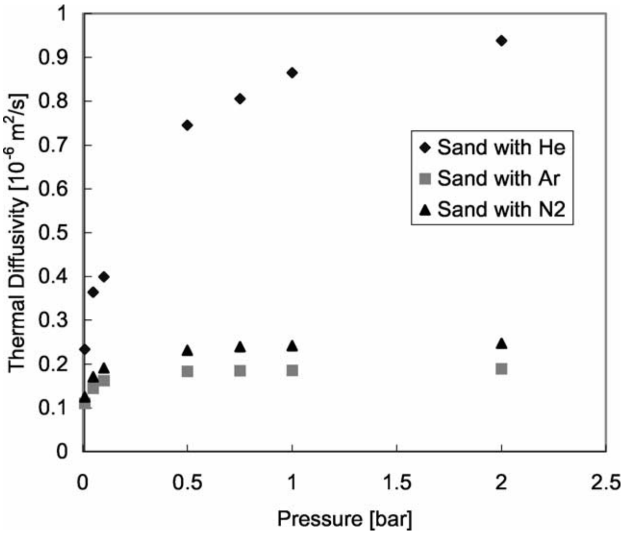

Fig. 6. Effect of filling gas and its pressure on the thermal diffusivity of synthetic sand packing.

The next important parameter to be investigated was the average temperature of the packing. To maintain the temperature at the desired level an additional heater (8) was used at the top of the test cell (Fig. 1). The experiments show a strong effect of temperature on the effective thermal diffusivity and relaxation time. Fig. 8 shows that the

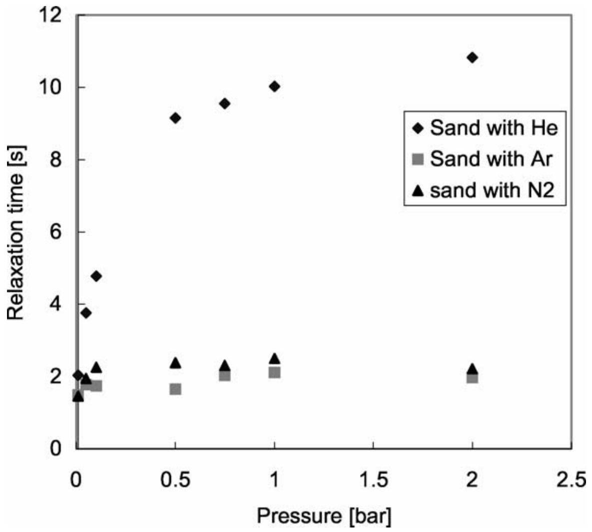

Fig. 7. Effect of filling gas and its pressure on the relaxation time of synthetic sand packing.

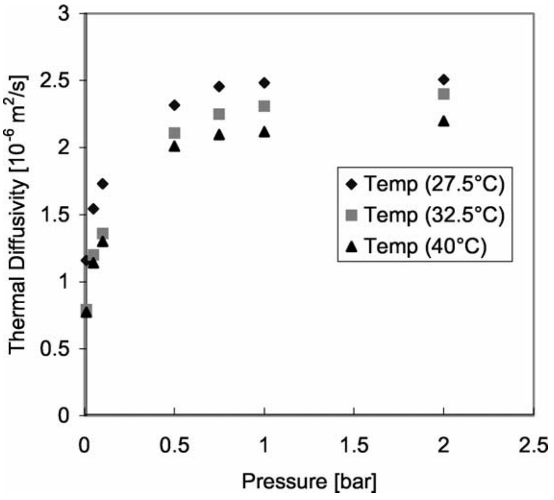

Fig. 8. Effect of temperature on the thermal diffusivity of Cu-Pb sphere packing with helium as the filling gas.

effective thermal diffusivity of packed bed of copper coated lead spheres with helium gas was decreased by 15 to 60% for temperature increase from 27.5 to 40 °C over the range of pressure 0.01 to 2.0 bar. Over the same range of temperature and pressure, the thermal relaxation time decreases by 10 to 30% as depicted in Fig. 9. The effect of temperature is expected in this line because the various effects, which composes the effective bulk thermal conductivity, are temperature dependent. The temperature effect is further investigated through experiment on sand with helium as the filling gas and a much smaller effect was observed. Figs. 10 and 11 show these effects on thermal diffusivity and relaxation time, respectively.

While investigating with packing of nanoparticles of Al_2O_3 (gamma) with an average particle size of 130 nanometers it was found that the effect of temperature on the thermal diffusivity is more than that of sand as shown in

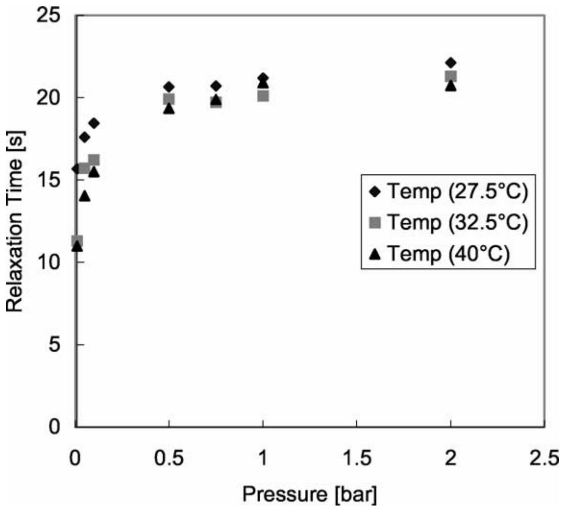

Fig. 9. Effect of temperature on the relaxation time of Cu-Pb sphere packing with helium as the filling gas.

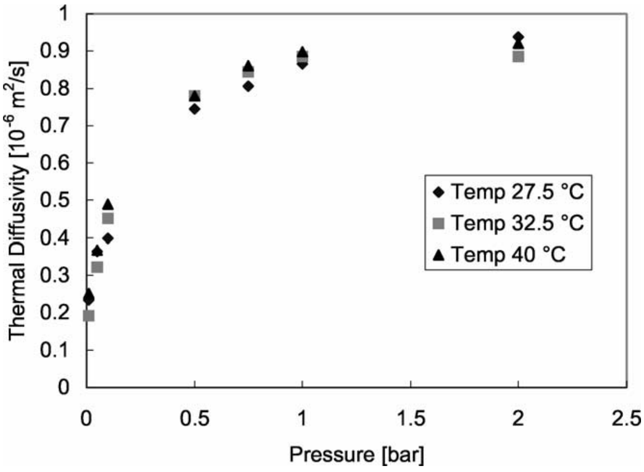

Fig. 10. Effect of temperature on the thermal diffusivity of synthetic sand packing with helium as the filling gas.

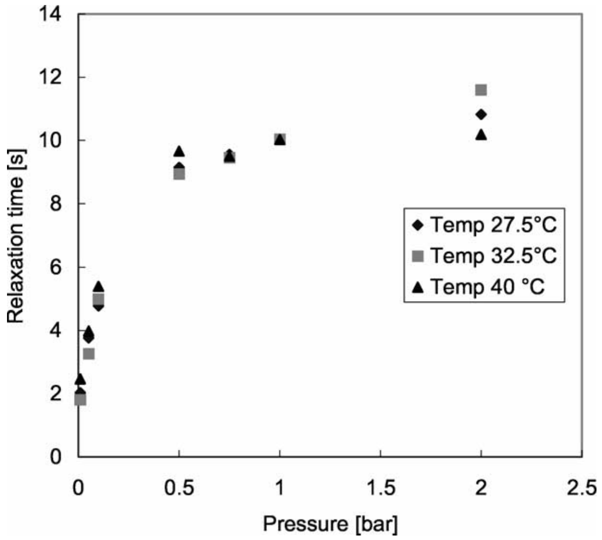

Fig. 11. Effect of temperature on the relaxation time of synthetic sand packing with helium as the filling gas.

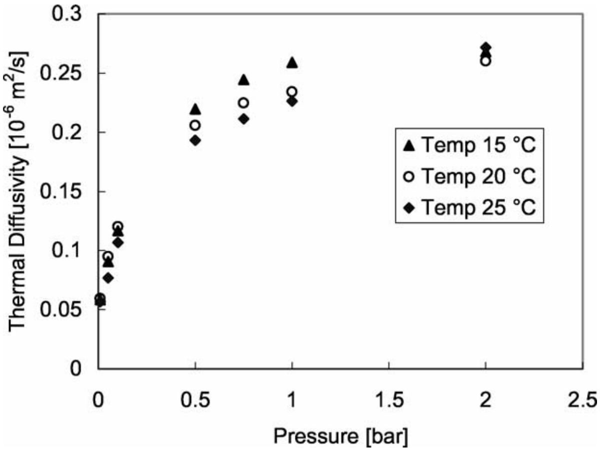

Fig. 12. Effect of temperature on the thermal diffusivity of Al_2O_3 nanoparticles with helium as the filling gas.

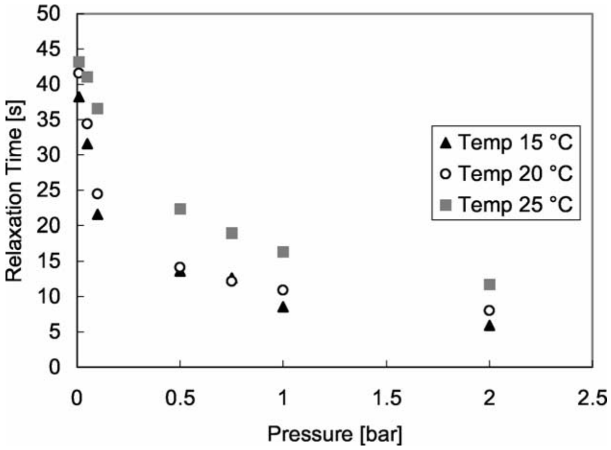

Fig. 13. Effect of temperature on the relaxation time of Al_2O_3 nanoparticles with helium as the filling gas.

Fig. 12. However, what was more surprising was the fact that in contrast to all the previous results, relaxation time decreased with pressure at the sub-atmospheric level as shown in Fig. 13. This forced the authors to look for the reason and it was found that in this case even though the relaxation time is decreased, the propagation velocity of second sound wave increased with pressure in a way similar to other cases. Fig. 14 shows the increase of propagation velocity of second sound wave with pressure for metallic sphere, sand and nanoparticle packing. This can be explained by the fact that higher pressure results in higher number of molecules resulting in number of intermolecular collisions which is the main mode of energy transport in gas giving a faster propagation of thermal wave.

Finally the effect of particle diameter was observed through experiments. To ascertain the effect of particle size spheres and powder of same Al_2O_3 (gamma) was used for experiment. The Al_2O_3 spheres of 6.2 mm diameter and nano powder Al_2O_3 with volume an averaged particle size of 130 nm have been used. For the nano size particles, the particle size distribution measured by dynamic light scattering

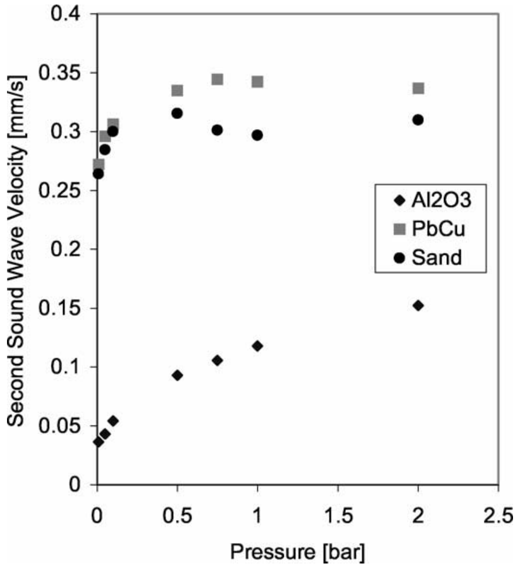

Fig. 14. Comparison of second sound velocity of different packings.

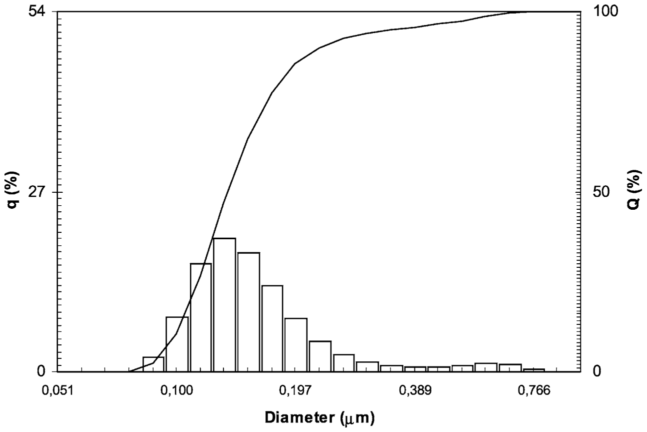

Fig. 15. Discrete and cumulative particle size distribution of nanoparticles.

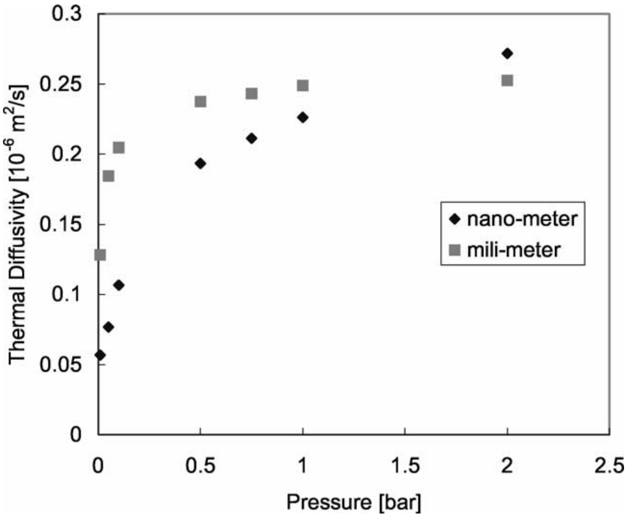

Fig. 16. Effect of particle size on the thermal diffusivity.

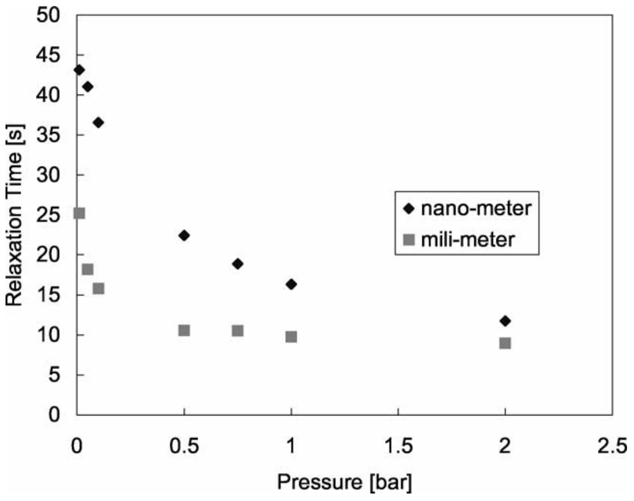

Fig. 17. Effect of particle size on relaxation time.

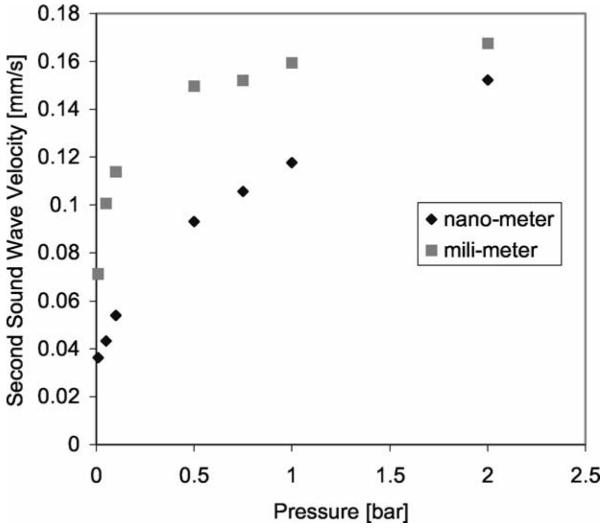

Fig. 18. Effect of particle size on second sound velocity.

technique is given in Fig. 15. The results given in Figs. 16 and 17 show that the thermal diffusivity increases with the increase in particle size but relaxation time decreases with it. The real picture again comes out through the computation of second sound velocity shown in Fig. 18 which shows an increase in particle size of Al_2O_3 presumably due to higher porosity and gas volume at higher particle size.

7. Conclusion

The present paper addresses one of the existing controversies in heat transfer, namely, the existence or otherwise of non-Fourier hyperbolic heat conduction in materials with non-homogeneous inner structure. Both claims and counter claims regarding hyperbolic nature of conduction existing in literature were shown to be inconsistent largely because of the flaw in experimental philosophy inducted by the respective investigators. The major limitation was found to be the independent determination of thermal diffusivity and relaxation time which goes against the basic premise of hyperbolic heat conduction based on a relaxation related constitutive equation for thermal diffusion. Based on this observation an experiment was devised to determine the thermal diffusivity and thermal relaxation time simultaneously from a single experiment using oscillatory temperature input in a semi infinite sample of the material under consideration. The experiment confirmed a hyperbolic behaviour of thermal propagation but smaller in extent compared to what has been reported in literature. To further widen the knowledge on this behaviour and to observe effect of different parameters such as filling gas and its pressure, particle size of solids and temperature of the packing a wide range of experimental results have been presented. It has been found that thermal diffusion behaviour and relaxation characteristics are influenced considerably by these parameters. Generally sub-atmospheric pressure have marked influence on the hyperbolic behaviour of the packing temperature and solid particle size also influence the hyperbolic behaviour significantly. However, it has been observed that the propagation velocity of second sound wave is a better indicator of hyperbolic behaviour rather than the usually used relaxation time which may lead to misinterpretation of experimental trends. The experimental results presented here can go a long way to settle the controversy on the existence of hyperbolic heat conduction in material with non-homogeneous inner structure. The authors feel that the present evidence of hyperbolic heat conduction in such material indicates the fact that in reality the various micro and macro level effects add up to result the effective bulk heat propagation which can be better modelled by hyperbolic law of conduction, which should not be confused with molecular conduction in homogenous media.

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